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Classical evolution of subspaces

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Abstract We study evolution of manifolds after their creation at high energies. Several kinds of gravitational Lagrangians with higher derivatives are considered. It is shown analytically and confirmed numerically that an asymptotic growth of the maximally symmetric manifolds depends strongly on their dimensionality. A number of final metrics describing our Universe is quite poor if we limit ourselves with a maximally symmetric extra space. We show that the initial conditions can be a reason of nontrivial solutions (funnels) and study their properties.

1 Introduction

The compact extra spaces is widely used idea. Their inclusion into physical theories helps to move forward on such issues as the grand unification [1,2], neutrino mass [3], the cosmological constant problem [4,5] and so on. Any multi-dimensional model has to lead to the effective 4-dim theory at low energies. This would imply relations between the observable four-dimensional physics and a metric of the higher dimensions.

One of the question remaining not clarified yet is: why specific number of dimensions are compactified and stable while others expand [6–8]? Which specific property of subspace leads to its quick growth? There are many attempts to clarify the problem, mostly related to introduction of fields other than gravity. It may be a scalar field [6,9] (most used case), gauge fields [10]. A static solutions can be obtained using the Casimir effect [11] or form fields [12,13]. Sometimes one of the subspace is assumed to be FRW space by definition [14]. Another possibility was discussed in [15,16]: it was shown that if the scale factor $a(t)$ of our 3D space is much larger than the growing scale factor $b(t)$ of the extra dimensions, a contradiction with observations can be avoided.

The origin of our Universe is usually related to its quantum creation from the space-time foam at high energies [17,18]. The probability of its creation is widely discussed, see e.g. [19]. Here we are interested in the subsequent classical evolution of the metrics rather than a calculation of this probability. Manifolds are nucleated having specific metrics. The set of such metrics is assumed to be very rich. After nucleation, these manifolds evolve classically forming a set of asymptotic manifolds, one of which could be our Universe. In this paper the asymptotic set of the maximally symmetric manifolds with positive curvature is studied in the framework of pure gravity with higher derivatives. We consider models of the $f(R)$ gravity and a more general model acting in 5 and 6 dimensions. No other fields are attracted to stabilize an extra space. We have found out that a number of asymptotic solutions is quite limited. This conclusion was confirmed both analytically and numerically. There is a set of initial conditions that lead to a common asymptote of classical solutions. In Sect. 3 we have elaborated a method for prediction the asymptotic behavior of metric judging on the specific form of the initial metric. We also study the funnel solution [20] as the result of an inhomogeneity of initial metric.

The gravity with higher derivatives is widely used in modern research despite the internal problems inherent in this approach [21]. Attempts to avoid the Ostrogradsky instabilities are made [22] and extensions of the Einstein–Hilbert action attract much attention. Promising branch of such models is based on the Gauss–Bonnet Lagrangian and its generalization to the Lovelock gravity. These models were adjusted to obtain differential equations of the second order so that the Ostrogradsky theorem is not dangerous for such models.

A lot of papers devoted to the $f(R)$-gravity – the simplest extension of the Einstein–Hilbert gravity. Reviews [23,24] contain description of the $f(R)$-theories including extension to the Gauss–Bonnet gravity. Examples of research with specific form of the function $f(R)$ can be found in [25,26]. Most of the research assume positive curvature of extra space metric, but as was shown in [27], hyperbolic manifolds can also be attracted to explain the observable acceleration of the Universe.

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